The elementary unit — canonical reviewer's comments on: Bureau International des Poids et Mesures (2019) The International System of Units (SI) 9th ed.

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Abbreviations: e: elementary charge; m_X: mass of a sample of X; M_{UX}: mass per elementary; n_X: amount of X; N: pure number; N_A: Avogadro constant; N_X: count of a sample of X; Q_u: quantity expressed in abstract units; Q_X: elementary quantity linked to a count; SI: International System of Units; u_Q: abstract unit of quantity Q_u or Q_X; U_X: elementary entity, material unit X; V_{UX}: volume per elementary; V_X: volume of a sample of X; x: elementary unit; X: entity type

Summary

"The International System of Units, the SI, has been used around the world as the preferred system of units, the basic language for science, technology, industry and trade since it was established in 1960." This statement heralds the 9th edition of the SI released on 2019-May-20. An new approach was introduced by defining the SI base units — and thus the abstract SI units in general — by their relation to fixed numerical values of fundamental constants of nature. Previous definitions of abstract units relied on a reference to concrete individual things realized as material artefacts, such as the International Prototype of the Kilogram (IPK). The (general) abstract unit 'kilogram' had to be calibrated in balance against an (individual) 'entetic' unit defining "1 kg", as a reference for the unit of mass and the mole [mol] as the unit of amount. Now the SI defines the mole as the fixed number of entities given by the Avogadro constant N_A. The elementary charge e is a fixed number of charges per proton. Amount and charge are thus in a fixed relation to the count of elementary entities U_X [x]. Count, amount, and charge are isomorphic elementary quantities. Amount and charge are linked to the count $N_X = N \cdot U_X$ with elementary unit x by fixed conversion constants N_A^{-1} [mol·x⁻¹] and e [C·x⁻¹], respectively. The SI does not use the elementary unit x. This causes a number of formal inconsistencies as discussed in the present communication on Euclid's unit, which is U_X, and Euclid's number, which is a count N_X.

Introduction

Thinking quantitatively is not easy outside very simply perceived and commonly encountered instances in every-day life. The number of individual persons in a family or individual collaborators in a company is a count, expressed by most societies and in most (but not all) languages in the same abstract unit that is understood intuitively and does not need an international convention. In contrast, the mass of the earth is beyond our perception. The mass of the International Prototype of the Kilogram (IPK) may appear as abstract as the unit kilogram itself, despite the fact that the IPK is a real artefact, a single individual thing, which is Euclid's definition of the unit. The International System of Units takes care of *abstract* units, which have their roots in *real*, material units, which are the defining realizations as artefacts or prototypes of a unit, such as the IPK. Some abstract units are far removed from our perception, such as the joule. There are scientists, who think of units of temperature as Celsius [°C] in the lab, but Fahrenheit [°F] at home. Several scientific journals recommend reporting data in SI units, but in the same journals we find respiration reported in time units of minutes [min], energy in units of calories [cal], and respiration as volume instead of molar amount of oxygen. This disconnects us from grasping quantitative terms — or do you have a quick recognition of the meaning 168.3 dozens of years? In 2020 we still must convince most authors and journals to consistently use SI units.

A problem is encountered in the International System of Units on the quantity 'count'. This accounts for substantial difficulties in general. How consistent are we in the formal use of the basic terms 'count' and 'number'? Can we refer to quickly recognizable disambiguous terms for quantities normalized for a count, such as mass, volume, or rate per count? In a position statement on quantitative 'Mitochondrial physiology', we did not follow SI guidelines on units of the quantity 'count', and had to find new ways for distinguishing the mass or volume of cells or organisms, versus mass or volume per single individual cell or organism (Gnaiger et al 2020). Whereas ad-hoc terms and symbols may be easily invented in a single publication or in practical language during a lecture, it is not easy to predict, if such invented terms are understood and properly decoded. Since even statisticians are quickly ready to present an apparently coherent story in view of few data (Kahneman 2011), I predict that misunderstandings of normalizations per count are the norm in some areas of the biomedical literature. Most importantly, in a general position statement, ad-hoc invented terms and symbols cannot be tolerated, but a consistent and coherent nomenclature is required with proper definitions. The difficulties may appear to be trivial, since counting and the unit linked to counting are not expected to present any significant problems. Counting is such a basic, fundamental, simple thing, deeply implemented in our common language. Did you realize the following terms used in this paragraph above: count, encounter, account? The fictitious 'Canonical reviewer's comments' on the probably most fundamental and formal publication in the scientific literature provides a better feeling than a theoretical treatise for a scientific readership to realize the actual problems and potential solutions offered in the publication BEC 2020.1. For the meaning of the term ‘canonical’ I refer to Hofstadter (1979). For the practical side of canonical pages, use numerical and canonical pages on opposite sides.

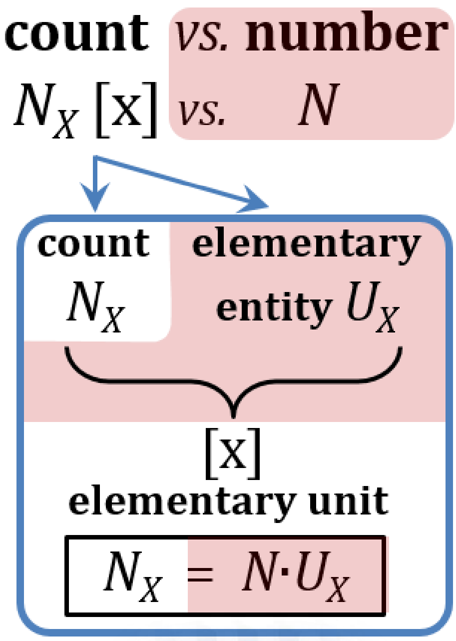
Canonical reviewer’s general comments

This SI publication is one of the most significant scientific, interdisciplinary and transdisciplinary publications of the century. Too many teachers and editors resist to making it a highly influential publication with actual impact on scientific publication, and

"The former International Prototype of the Kilogram (IPK) is an artefact whose mass defined the SI unit of mass until the implementation of a revised definition of the kilogram on 20 May 2019. .. The unit of mass is disseminated throughout the world by comparisons with the IPK made indirectly through a hierarchical system of mass standards. Historically the IPK has been compared to its official copies at intervals of about 40 years, with the exception of the "extraordinary campaign" carried out in 2014, which was only 22 years after the preceding one. In the intervals in between, the working standards are used to disseminate the kilogram unit to the Member States by calibrations of their "national prototypes", which are intended to serve as national standards." — https://www.bipm.org/en/bipm/mass/ipk/

Count N_X [x]

Count N_X is the number N of elementary entities of entity-type X. The single elementary entity U_X is a countable object or event. N_X is the number of objects of type X, whereas the term 'entity' and symbol X are frequently used and understood in dual-message code indicating both (*1*) the entity-type X and (*2*) a count of N_X = 1 x for a single elementary entity U_X. 'Count' is synonymous with 'number of entities' (number of particles such as molecules, or objects such as cells). Count is one of the most fundamental quantities in all areas of physics to biology, sociology, economy and philosphy, including all perspectives of the statics of countable objects to the dynamics of countable events. The term 'number of entities' can be used in short for 'number of elementary entities', since only elementary entities can be counted, and as long as it is clear from the context, that it is not the number of different entity types that are the object of the count.



thus contribute to the current communication crisis. This does not reflect any faults of the SI authors but a lack of governance to improve scientific communication, and the limitations of the non-SI readers trapped into the International System of Impact Factors rather than guided towards implementing the International System of Units. What is the numerical value and the practical meaning of the Impact Factor of this SI publication?

Quantity values being pure numbers (Section 5.4.7 Stating quantity values being pure numbers; p. 151):

> "As discussed in Section 2.3.3, values of quantities with unit one, are expressed simply as numbers. The unit symbol 1 or unit name “one” are not explicitly shown. SI prefix symbols can neither be attached to the symbol 1 nor to the name “one”, therefore powers of 10 are used to express particularly large or small values."

What are — apart from ratios of quantities — these "quantities with unit one"?

The quantity count

Section 2.3.3 Dimensions of quantities: On p. 136, the quantity **count** is mentioned:

> "All other quantities, with the exception of counts, are derived quantities, which may be written in terms of base quantities according to the equations of physics. .. There are also some quantities that cannot be described in terms of the seven base quantities of the SI, but have the nature of a count. Examples are a number of molecules, a number of cellular or biomolecular entities (for example copies of a particular nucleic acid sequence), or degeneracy in quantum mechanics. Counting quantities are also quantities with the associated unit one."

Importantly and surprisingly, a general definition of the quantity *count* is missing. This is particularly disconcerting, since it appears to ignore the specific emphasis placed on clarification on p. 137:

- "It is especially important to have a clear description of any quantity with unit one (see section 5.4.7) that is expressed as a ratio of quantities of the same kind (for example length ratios or amount fractions) or as a count (for example number of photons or decays)."
- In section 5.4.7 (p. 151): "Quantities relating to counting do not have this option {of being expressed with units (m/m, mol/mol)}, they are just numbers."
- On p. 140: "the SI unit of activity is becquerel, implying counts per second".

These phrases are the only places where count(s) and counting are mentioned in the SI document, besides 'counting fringes' (p. 207: ".. using an interferometer with a travelling microscope to measure the optical path difference as the fringes were counted") and p. 129.

Unit of count

On p. 129 a "quantity for counting entities" is given as N_A times amount of substance.

> "The Avogadro constant N_A is a proportionality constant between the quantity amount of substance (with unit mole) and the quantity for counting entities (with unit one, symbol 1)."

A unit should not be a number, and the symbol for a unit should not be a numeral. In Resolution 1 (p. 189) a consensus is reached on the importance of "a redefinition of a number of units of the International System of Units (SI)".

The definition of *count* and redefinition of the *unit of count* remain an open task.

PHILOSOPHY OF NUMBER

MARCUS GIAQUINTO

THERE are many kinds of number: natural numbers, integers, rational numbers, real numbers, complex numbers and others. Moreover, the system of natural numbers is instantiated by both the finite cardinal numbers and the finite ordinal numbers. We cannot deal properly with all of these number kinds here. This chapter concentrates on the finite cardinal numbers. These are the numbers which are answers to questions of the form 'How many Fs are there?' In what follows, an unqualified use of the word 'number' abbreviates 'cardinal number'.

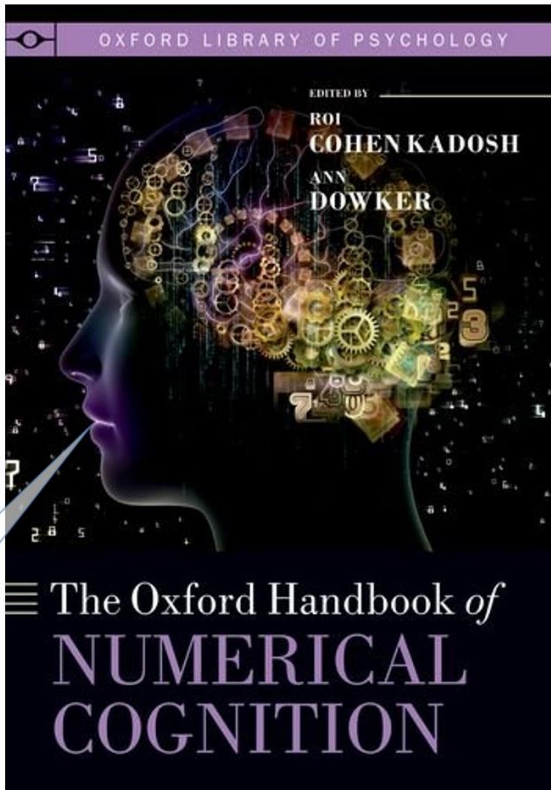
Numbers cannot be seen, heard, touched, tasted, or smelled; they do not emit or reflect signals; they leave no traces. So what kind of things are they? How can we have knowledge of them? These are the central philosophical questions about numbers. Plausible combinations of answers have proved elusive. The aim of this chapter is to present and assess the main views – classical and neo-classical, nominalism, mentalism, fictionalism, logicism, and the set-size view. All views are disputed, including the view I will argue for – the set-size view. The final section relates finite cardinal numbers to natural numbers.

THE CLASSICAL VIEW: MULTITUDES OF UNITS

At the start of Book VII of *Euclid's Elements*, having defined a 'unit' to be a single individual thing, a number (arithmos) is defined thus:

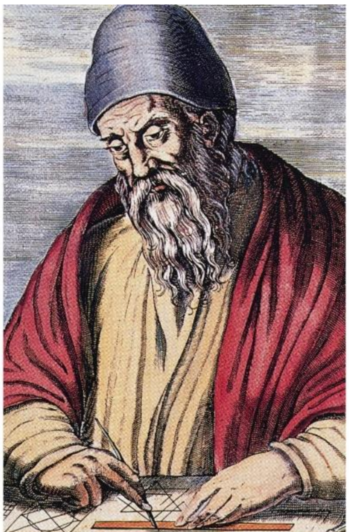
A number is a multitude of units.

The Euclidean unit is great!
But 'number' – should it rather be
'A count is a number of units'?



Euclid's unit is the entetic unit: the single individual thing.

Euclid's number is the count: the number of Euclidean units.



Euclid of Alexandria, c. 300 BC, coloured woodcut, 1584

Elements

MitoPedia

Number *N*

A **number** *N* (or *n*) is a count N_X [x] divided by the elementary entity U_X [x]. *X* must represent the same entity in both occurences. The elementary unit [x] cancels in the division by simplification, such that numbers (for example, numbers 8 or 24) are abstracted from the counted entity *X*.

Technical comments

Spelling

Since 'small spelling variations occur in the language of the English speaking countries (for instance, "metre" and "meter", "litre" and "liter")' (p. 124), a decision should be taken for consistent spelling in a document. The English text of the SI brochure follows the style "metre" and "litre". It is found that in the scientific literature the spelling style "meter" and "liter" prevails even in European journals. In the following quotes from the SI brochure (with reference to page numbers in the 9th edition), the spelling style is changed, which does not exert any influence on the meaning.

Quantity calculus (p. 148)

> "Symbols for units are treated as mathematical entities. In expressing the value of a quantity as the product of a numerical value and a unit, both the numerical value and the unit may be treated by the ordinary rules of algebra. This procedure is described as the use of quantity calculus, or the algebra of quantities. For example, the equation p = 48 kPa may equally be written as p/kPa = 48. It is common practice to write the quotient of a quantity and a unit in this way for a column heading in a table, so that the entries in the table are simply numbers."

Suggestion: p_{O2}/[kPa] = 18.6; S_{O2}/[µmol·kPa^{-1}] = 9.72

Without changing any of the above rules, it is useful to put units into brackets to designate the units specifically in quantity calculus.

No comment

Defining the unit of a quantity (p. 127)

> "The value of a quantity is generally expressed as the product of a number and a unit. The unit is simply a particular example of the quantity concerned which is used as a reference, and the number is the ratio of the value of the quantity to the unit.
>
> "For example, the speed of light in vacuum is a constant of nature, denoted by c, whose value in SI units is given by the relation c = 299 792 458 m/s where the numerical value is 299 792 458 and the unit is m/s.
>
> "For a particular quantity different units may be used. For example, the value of the speed v of a particle may be expressed as v = 25 m/s or v = 90 km/h, where meter per second and kilometer per hour are alternative units for the same value of the quantity speed.
>
> "Before stating the result of a measurement, it is essential that the quantity being presented is adequately described. This may be simple, as in the case of the length of a particular steel rod, but can become more complex when higher accuracy is required and where additional parameters, such as temperature, need to be specified.
>
> "When a measurement result of a quantity is reported, the ***estimated*** value of the measurand (the quantity to be measured), and the ***uncertainty*** associated with that value, are necessary. Both are expressed in the same unit."

> *Numbers cannot be seen, heard, touched, tasted, or smelled; they do not emit or reflect signals; they leave no traces. So what kind of things are they? These are the central philosophical questions about numbers. Plausible combinations of answers have proved elusive.* — Marcus Giaquinto, in Kadosh 2015 Oxford Univ Press

Entity *X*

Theoretically, everything can be counted with some uncertainty in the microscopic world of quantum physics. A sample of Christmas pudding can be counted in terms of atoms — theoretically. Practically, there are non-countable entities *X*, such as a Christmas pudding. It does not have natural, recognizable individual parts that can be counted (except for the two spherical decorations on top of it).

But the pudding seen as a serving on a dish has been partitioned from a larger mass of pudding. The servings are partitioned according to an external concept of a serving unit, in contrast to an internal pattern of replicative items. After artificial partitioning and serving on plates of the entity type *X* = pudding, we can recognize the samples of pudding on the plates as countable entities. Then our sample is not a single big or small mass of pudding (such as the mass of pudding in the picture, which cannot be quantified in terms of countable parts), but the new and larger sample is the assembly of servings of partitioned pudding prepared for the entire *X*-mass party. In this sample the single individual — and hence countable — thing is defined as the single serving, without any reference to its parts. This type of sample consists of a number of servings, and the servings are the countable parts of the sample, the units of pudding served and enjoyed at the *X*-mass party. Talking about stone, units of stone (stones or puddings versus stone or pudding) are even harder to partition and to bite than *X*-mass pudding.

Canonical comments

Definition of the SI (p. 127)

1. "As for any quantity, the value of a fundamental constant can be expressed as the product of a number and a unit."
2. "The definitions below specify the exact numerical value of each constant when its value is expressed in the corresponding SI unit. By fixing the exact numerical value the unit becomes defined, since the product of the numerical value and the unit has to equal the value of the constant, which is postulated to be invariant."

Comment: The terms 'number' and 'numerical value' are used as being equivalent in the two phrases above: (1) value of a fundamental constant = "product of a number and a unit"; (2) value of the constant = "product of the numerical value and the unit".

It should be considered to define: The value of a quantity Q (Q_X or Q_u) is the product of the **numerical value** of a number N and a unit u_Q. Symbols for quantities Q_X are, e.g. N_X and n_X for count and amount, respectively; symbols for quantities Q_u are, e.g. m and V for mass and volume, respectively. Do these symbols represent merely the quantity *type*? Interpret a formula such as m = 60 kg: The symbol m represents the quantity 'mass', the numerical value of the number N is 60 (N = 60), the unit is u_m = kg, and the value of the quantity m is 60 kg. Just in case that these definitions appear to be acceptable, then it follows: quantity m = value of the quantity m. The number N is not a quantity and has no units. The numerical value of a number N is a pure number. N is the symbol for any number, and 60 is the numeral for the number with numerical value of 60 = 6·10.

"The seven constants are chosen in such a way that any unit of the SI can be written either through a defining constant itself or through products or quotients of defining constants.

The International System of Units, the SI, is the system of units in which

- **the unperturbed ground state hyperfine transition frequency of the caesium 133 atom $\Delta\nu_{Cs}$ is 9 192 631 770 Hz,**
- **the speed of light in vacuum *c* is 299 792 458 m/s,**
- **the Planck constant *h* is 6.626 070 15 × 10^{-34} J s,**
- **the elementary charge *e* is 1.602 176 634 × 10^{-19} C x^{-1},**
- **the Boltzmann constant *k* is 1.380 649 × 10^{-23} J x^{-1} K^{-1},**
- **the Avogadro constant N_A is 6.022 140 76 × 10^{23} x mol^{-1},**
- **the luminous efficacy of monochromatic radiation of frequency 540 × 1012 Hz, K_{cd}, is 683 lm/W,**

where the hertz, joule, coulomb, lumen, and watt, with unit symbols Hz, J, C, lm, and W, respectively, are related to the units second, meter, kilogram, ampere, kelvin, mole, and candela, with unit symbols s, m, kg, A, K, mol, and cd, respectively, according to Hz = **x** s^{-1}, J = kg m^2 s^{-2}, C = A s, lm = cd m^2 m^{-2} = cd sr, and W = kg m^2 s^{-3}.

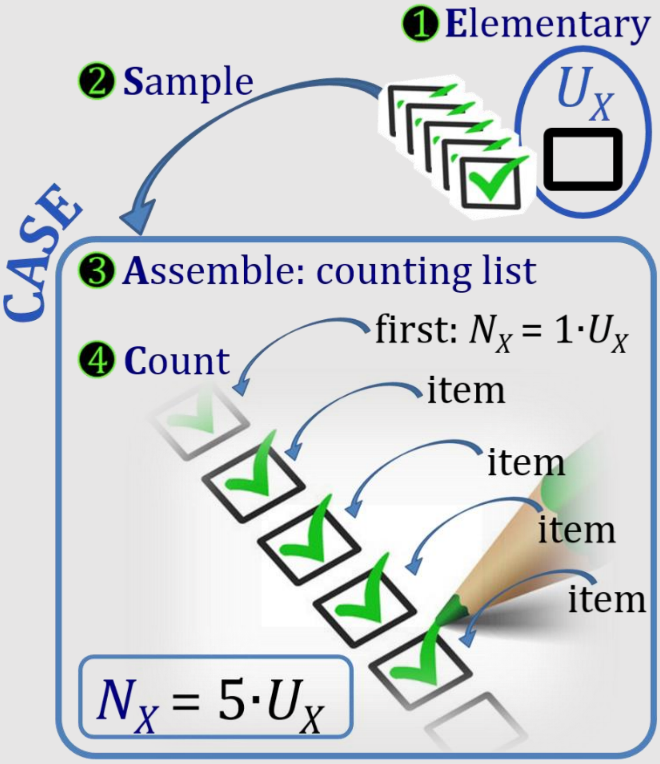
The numerical values of the seven defining constants have no uncertainty."

The nature of the seven defining constants (p. 129)

"The Avogadro constant N_A is a proportionality constant between the quantity amount of substance (with unit mole) and the quantity for counting entities (with unit one, symbol 1)."

The Euclidean unit U_X and items

Counting can begin only after defining the single individual thing (the Euclidean unit). Having defined the countable unit X as the **E**lementary entity U_X externally as step 1, the next step 2 is **S**ampling as the transfer into a counting system, followed by step 3 of **A**ssembling into a counting list, and the last step 4 is **C**ounting, starting with the first elementary entity in the counting list ($N_X = 1 \cdot U_X$) and adding sequentially item after item. In this case, you should read **CASE** backwards. The term '**item**' derives from the Latin word *item* meaning 'also, moreover, likewise', and the present meaning in English stems from adding identical objects after the first one into a list, item after item. Since there was no counting before counting begins (ESAC), the single individual thing has not yet been counted — U_X is entirely different from a count of one. And with a single individual X-mass pudding on the menu, there is no X-mass party on the counting table — in the counting table.



Comment: Comparing the terms 'quantity amount of substance (with unit mole)' and 'quantity for counting entities (with unit one, symbol 1)' raises several questions that lead to the following comments:

(*1*) For 'quantity for counting entities' the proper name should be given, comparable to *amount*. The proper name is *count*.

(*2*) The term 'quantity for counting entities' is ambiguous: Is it a quantity for counting? Is it a quantity for entities that are counting (entities such as counting machines, cell counters, ticket counters)?

(*3*) Unit one, symbol 1: This is a profound mix-up of a numerical value or number 'one' (numeral 1) with a *unit*. There is an inconsistency compared to all other units in the SI, since the 'one' (numeral 1) could be added to any other SI unit (or not explicitly shown), e.g. volume per mass V/m with unit [m^3]/[1 kg] instead of [m^3]/[kg], compared to volume per count V/N_X with unit [m^3]/[1 x] instead of [m^3]/[x] — not [m^3]/[1] or [m^3].

Quantity symbols and unit symbols (p. 149)

> "Unit symbols must not be used to provide specific information about the quantity and should never be the sole source of information on the quantity. Units are never qualified by further information about the nature of the quantity; any extra information on the nature of the quantity should be attached to the quantity symbol and not to the unit symbol."

Comment: "Units are never qualified by further information about the nature of the quantity" — this generalization is valid only for units restricted to the class of *abstract units* u_Q. Units u_Q are abstracted from the nature of the entity X which is expressed as a quantity Q_u. In contrast, it is essential to qualify any Euclidean unit U_X by X as the defining 'single individual thing': the unit particle, the unit atom O, the unit O_2 molecule, the unit endothelial cell, the unit organism of type org, the unit event of type X.

The term "nature of the quantity" is ambiguous, with potentially dualistic meaning as (*1*) the nature of the quantity being mass, length, volume, amount of substance, etc.; or (*2*) the nature of entity X of the quantity, meaning a volume of O_2 versus CO_2, a mass of pudding or stones, etc. Did SI intend the second interpretation, or both?

Quantity calculus can be complemented by providing specific information about the entity (e.g. entity O_2) separate from the unit symbol (not attached to the unit symbol), but subsequent to the unit symbol, e.g. [mol] O_2 or [kJ·mol^{-1}] O_2, such that the entity-type is not presented in the place occupied by the unit symbol for division or multiplication, respectively.

Writing and printing of unit symbols and of numbers — Resolution 7 (p. 162)

> "Roman (upright) type, in general lower-case, is used for symbols of units; if, however, the symbols are derived from proper names, capital roman type is used. These symbols are not followed by a full stop."

An 'elementary entity' is the *real unit* or Euclidean unit, which is tightly connected to a concrete object, to an individual thing (a single individual serving of pudding, a single block of stone, a single individual body). Thus the Euclidean unit is *realized* in the real world, and mentally *realized* as a formal quantity in a formal system of quantities and units, in the cognitive sequence of counting before counting can begin.

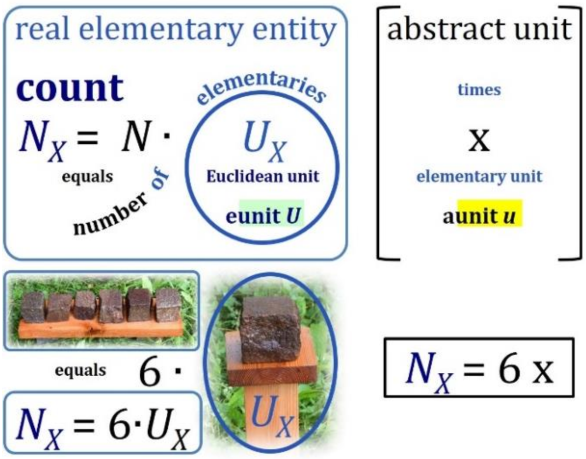
Formalization requires the creation of a symbol coherent with the system of symbols for quantities and units. The SI symbol X for entity is not appropriate as the symbol for elementary entities, since not all entities X are in fact countable. The symbol of a count of entities X is N_X, which is absolutely inappropriate as the symbol for elementary entities, since the elementary entity has to be defined as a quantity *before* counting can begin. The elementary entity, therefore, is not a count N_X. The symbol U_X is given to the quantity 'elementary entity' (or unit X; Gnaiger 2020; Gnaiger et al 2020), to emphasize the *single thing*, the *real unit* of specified entity type X. On the other hand, the *abstract unit* kilogram [kg] expresses the quantity mass m of any kind of sample, in which the type of physical object does not even have to be known, because abstract units are detached from any real thing.

A kg of pudding is a kg, independent of the recipe and ingredients, and a kg of body mass is independent of composition in terms of carbohydrate, protein and fat. If you carry too much fat or too many stones in your pocket, you have a body mass excess, albeit with different meanings. Any *abstract unit* ("aunit", e.g. kilogram, meter, liter, joule; with symbols kg, m, L, J) relates to *anything* that is measured as opposed to counting. In contrast, the Euclidean *entetic unit* ("eunit") relates to a specific *thing* that can be counted — a countable entity. Comparable to all quantities that are expressed in abstract units (mass, height, volume, energy), the quantity 'elementary entity' U_X as a *real unit* (unit of type X; eunit) is expressed in the *abstract unit* (aunit) with the name 'elementary unit' and symbol x. The meaning of this symbol [x] is, that it has the numerical value of one, independent of the nature of X that defines the elementary entity U_X.

Elementary U_X [x]

An **elementary entity** is an entity of type X, distinguished as a single *unit* of countable objects (X = molecules, cells, organisms, items) or events (X = beats, collisions, emissions, decays, celestial cycles, instances, occurrences, parties). An elementary entity needs to be distinguished from non-countable entities and the general class of entities X, as emphasized by the term 'elementary' (synonymous with 'elementary entity') with symbol U_X and elementary unit [x].

A number of defined elementaries U_X is a count, $N_X = N \cdot U_X$ [x], where N is a number, and as such N is dimensionless. N is a *number* (stop) and is not 'a number of ..'. Elementaries are added as items to a count. The elementary U_X has the dimension U of the count N_X. U_X gives the count the defining 'counting-unit', which is the 'elementary unit' [x]. Count divided by elementary is a pure number, $N = N_X \cdot U_X^{-1}$. The unit x of a count can neither be the entity X nor a number. Since a count N_X is the number of elementary entities, the elementary U_X is not a count (U_X is not identical with $N \cdot U_X$).



Comment: The SI symbol '1' suggested for the unit of the quantity *count*, does not follow this SI procedure. A symbol for the unit of count is required, that is consistent with SI Resolution 7.

> "In numbers, the comma (French practice) or the dot (British practice) is used only to separate the integral part of numbers from the decimal part. Numbers may be divided in groups of three in order to facilitate reading; neither dots nor commas are ever inserted in the spaces between groups."

Comment: In the last sentence above, there is a confusion between **numbers**, **numerals** (representing numbers, such as 4, 12, 5093.78, 6, in a specific numeral system), and **digits** (or **characters**, such as the ten characters in the decimal numeral system: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9). When a large **number** is expressed by a **numeral** as a string of several symbols, the **digits** may be separated in groups of three ..

For comparison, IUPAC presents the same message in the following correct form: "To facilitate the reading of long numbers the digits may be grouped in threes about the decimal sign but no point or comma should be used except for the decimal sign."

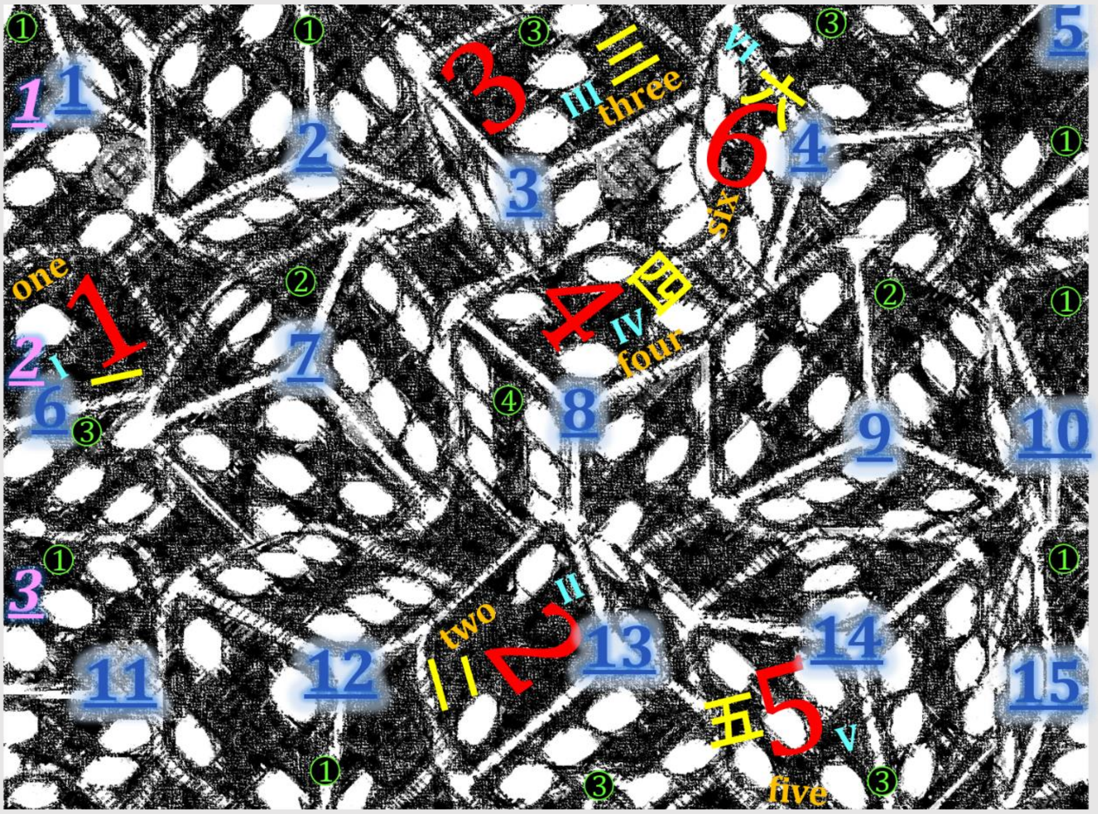
The historical development of the realization of SI units (p. 204)

> ".. the masses of a silicon atom (averaged over the three isotopes used for the sphere) m_{Si}, and the electron m_{e} .."

Comment: Defining entity X as Si, the SI symbol for the mass of a sample of Si is m_{Si} [kg]. From a count N_{Si} of Si in the sample, the mass per silicon atom is $m_{\mathrm{Si}}\cdot N_{\mathrm{Si}}^{-1}$ [kg·x^{-1}]. The same holds for the mass per any elementary entity of entity-type X. For consistency, the general term 'mass of an entity' ("masses of a silicon atom .. and the electron") has to be replaced by the general term 'mass per elementary entity' as proposed by Gnaiger et al 2020 ("masses *per* silicon atom .. and *per* electron"). The symbol m_{Si} cannot be used for the expression $m_{\mathrm{Si}}\cdot N_{\mathrm{Si}}^{-1}$. This elementary, formal inconsistency must be resolved in the SI to achieve coherence. Elementary mass per Si atom is $M_{U\mathrm{Si}} = m_{\mathrm{Si}}\cdot(N\cdot U_{\mathrm{Si}})^{-1}$ [kg·x^{-1}].

Unit of amount of substance, mole (p. 209)

> "The quantity used by chemists to specify the amount of chemical elements or compounds is called “amount of substance”. Amount of substance, symbol n, is defined to be proportional to the number of specified elementary entities N in a sample, the proportionality constant being a universal constant which is the same for all entities. The proportionality constant is the reciprocal of the Avogadro constant N_{A}, so that $n = N/N_{\mathrm{A}}$. The unit of amount of substance is called the *mole*, symbol mol. Following proposals by the IUPAP, IUPAC and ISO, the CIPM developed a definition of the mole in 1967 and confirmed it in 1969, by specifying that the molar mass of carbon 12 should be exactly 0.012 kg/mol. This allowed the amount of substance $n_{\mathrm{S}}(X)$ of any pure sample S of entity X to be determined directly from the mass of the sample m_{S} and the molar mass $M(X)$ of entity X, the molar mass being determined from its relative atomic mass A_{r} (atomic or molecular weight) without the need for a precise knowledge of the Avogadro constant, by using the relations
>
> $$n_{\mathrm{S}}(X) = m_{\mathrm{S}}/M(X), \text{ and } M(X) = A_{\mathrm{r}}(X) \text{ g/mol}$$
>
> Thus, this definition of the mole was dependent on the artefact definition of the kilogram.



Formats and meanings of numbers

1. Counting and notation types: (1.1) dice, (1.2) Roman numerals, (1.3) Mandarin-Chinese signs, (1.4) Arabic numerals, (1.5) English number-words. The dice format requires hardly any interpretation, since the signal for counting is given in a series of linear expansion; this works well up to :::, but does not work for 66 or 666. Similarly, Roman and Mandarin symbols from I to III do not need interpretation due to the signal for counting, but IV to VI is more complex in Roman and Mandarin notations by compression required for extension towards higher numbers. Interpretation of Arabic numerals and English words needs learning from beginning with 1 and one, since the formal relation to counting is abandoned in favor of reduction; these investments pay off in the long run — once Arabic numerals have been learned, these symbols can be recognized and distinguished most rapidly, be written most economically, and be extended to high numbers by combination in the decimal number system. English words are much less economical in writing, but they connect isomorphically the image of the written number-word with the acoustic form of the spoken number-word.

2. Cardinal counting and ordinal ranking of dice: There are 15 dice in the figure. Dice 1 to 5 are in row *1*; dice 6 to 10 are in row *2*; dice 11 to 15 are in row *3*.

> The numerical value of the Avogadro constant defined in this way was equal to the number of atoms in 12 grams of carbon 12. However, because of recent technological advances, this number is now known with such precision that a simpler and more universal definition of the mole has become possible, namely, by specifying exactly the number of entities in one mole of any substance, thus fixing the numerical value of the Avogadro constant. This has the effect that the new definition of the mole and the value of the Avogadro constant are no longer dependent on the definition of the kilogram."

Comment 1: Clarification is required to explain the difference between a *number* and a "number of specified elementary entities N in a sample". If symbol N is used to represent a 'number', then the same symbol N should not be used to represent a 'number of specified elementary entities'. The term 'number' is used in defining the unit of a quantity (Section 2.1): "The value of a quantity is generally expressed as the product of a number and a unit." Distinguishable symbols should be used for "number of entities" N_X with elementary unit [x] and dimensionless "number" N (without subscript; Gnaiger et al 2020). It might be appropriate, to use the lower-case n for dimensionless number, but this is not practical, since it might be confused with the symbol n for amount, as in $n_X = N_X/N_A$.

Comment *2*: The term *entity* (*entities*) is used with two different meanings. Whereas these meanings can be understood by decoding in context using practical language, this ambiguity should be avoided in a formal system of terminology: (*i*) In the context "amount of substance $n_S(X)$ of any pure sample S of entity X", and "molar mass $M(X)$ of entity X", the term *entity* and symbol X are used with the meaning *entity-type*. (*ii*) If this interpretation in terms of specification of entity-type is taken rigorously, then the term "by specifying exactly the number of entities in one mole of any substance, thus fixing the numerical value of the Avogadro constant" must be understood as indicating, that in one mole of any such substance there are N_A different entity-types. The intention of this comment is not to suggest, that anybody should make such a non-sensical interpretation, but rather to point to the formal inconsistency of the terminological system. The SI lacks a distinction between the term *entity-type* X (which does not express any quantity) and the *elementary entity* U_X (which is an 'external' elementary quantity that needs to be defined before counting can start with a count N_X = 1 x). With this clarification in mind, it then makes sense to use practical language: (*i*) "sample S of entity X"; (*ii*) "number of elementary entities" = count, $N_X = N \cdot U_X$ with elementary unit [x] (Gnaiger et al 2020).

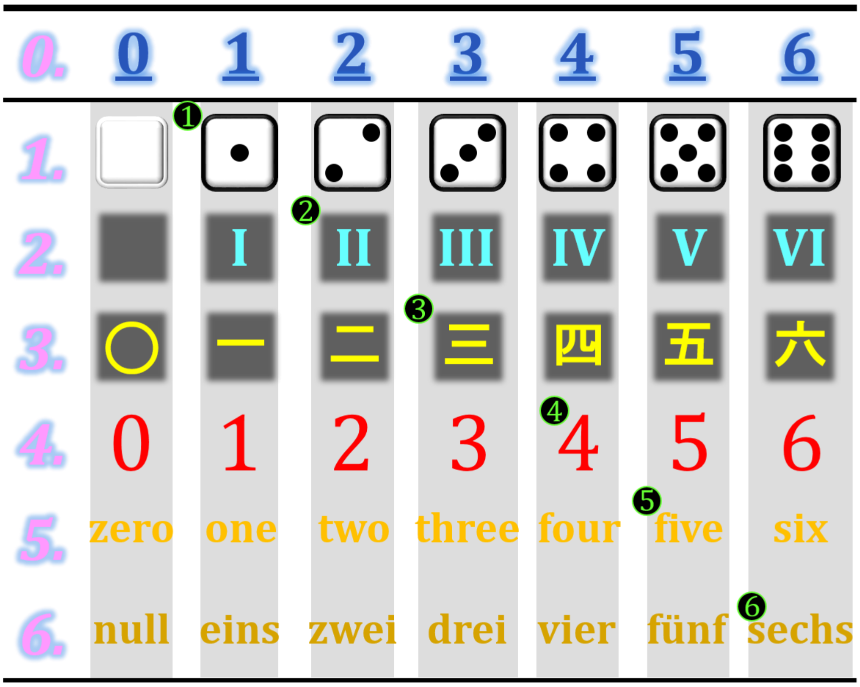
Comment 3: The symbols $n_S(X)$ and m_S are well defined, such that the meaning of the message can be understood. There remains, however, the difficulty to understand the logic of selecting these symbols. The amount $n_S(X)$ [mol] of X can be calculated from the mass m_S [g] of pure sample S and the molar mass $M(X)$ [g·mol^{-1}] of entity X only, if there is a pure sample S of entity X. Why then is this essential information not added to the symbol $m_S(X)$, comparable to $n_S(X)$?

Comment 4: If m_S or $m_S(X)$ [g] is the symbol for the mass of a pure sample of entity X, and $n_S(X)$ [mol] is the symbol for amount of substance of a pure sample S of entity X, are these then equivalent to m_X [g] and n_X [mol], respectively, as the symbols for the mass and amount of entity X? Can a mass or amount be determined for an entity X that is not sampled? It may be clarified that a *quantity* by definition can be measured (mass) or counted (amount) only in a defined sample of a defined entity X. It should be noted that "defined entity X" may be something rather undefined: A number of trees can be sampled;

3. Nominal labelling: Dice of tye ① are with single notation and positioned on the marging of the figure; ② dice with single notation and positioned in the center of the figure; ③ dice with multiple notations and positioned on the margin of the figure; ④ dice with multiple notations and positioned in the center of the figure.

4. Number magnitude and space: Dice with different notation types have a numerical magnitude that increases with their position in space from left to right (1 to 6). This spatial association is less pronounced for Mandarin notation type.

5. Sex of numbers and numerical parity: Even numbers such as 6 are associated with female sex, and are likened more than odd numbers such as 3, which connotate masculinity (Wilkie 2015 Front Psychol). Even odd and even numbers are gendered. Isn't it odd to be the odd man?



Numbers — numerals — notations: ***1.*** Symbols for 1 to 6 on dice showing figures of dots (pips) as *cardinal* numerals for counting. Roman numerals have the same problem with the number 0 as a die. ***2.*** Lines are shown for counting-numerals as iconic symbols I, II, and III, comparable to the dots on a die. ***3.*** The Chinese Mandarin includes zero as a number ○, and shows counting-numerals as lines comparable to the Roman iconic counting-numerals I, II, and III. ***4.*** The Arabic number system adopted the zero from Asian Indian mathematicians. Cipher (German *Ziffer*) is rooted in the Arabic *sifr*, which stems from India for *zero*. The Arabic numeral 1 can be seen as an iconic symbol or counting-numeral comparable to the Roman I and Chinese —. ***5.*** English number-words. ***6.*** German number-words.

the mass of these trees can be measured; the number of trees (converted from count to amount) that are contained in the sample may or may not be known; the species of trees may be a pine or any other mixture of trees; the number of pinecones and needles may or may not be determined. In contrast to this potentially but not necessarily vague definition of entity *X*, the sample must be well defined in the sense of being separated from the rest of the world for measuring or counting, otherwise a *quantity* does not make sense. Therefore, the symbols $n_S(X)$ and $m_S(X)$ do not make logical sense and should be replaced by n_X and m_X.

Comment 5: The symbol $M(X)$ is well defined to indicate the molar mass $M(X)$ [g·mol^{-1}] of entity *X*, but there remains a problem with an extension to obtain a consistent system of symbols for other derived quantities, when mass is normalized not only for amount [g·mol^{-1}] but count [g·x^{-1}] and volume [g·L^{-1}] (or for the SI [kg·m^{-3}]). In different contexts there are *ad hoc* practical symbols in use, but a formally consistent system of symbols does not exist, as shown in **Table 1**, comparing canonical and practical symbols for harmonization.

Figures 1 and 2 compare elementary quantities that are stoichiometrically linked to counting of elementary entities with extensive quantities based on measurements of a pure sample. Count, amount, and charge depend on the definition of elementary entities, whereas measurements of extensive quantities are independent of information on discrete entetic units. Red-shaded terms and units in **Figure 2** are not implemented in the SI. Entity *X* is used strictly in the sense of *entity-type X*. In practical language, however, the entity-type *X* = *body* in the context of human and animal bodies is understood as indicating the elementary entity in terms of a single individual body, linked to the perception, that body mass is measured routinely on a sample of a single individual organism.

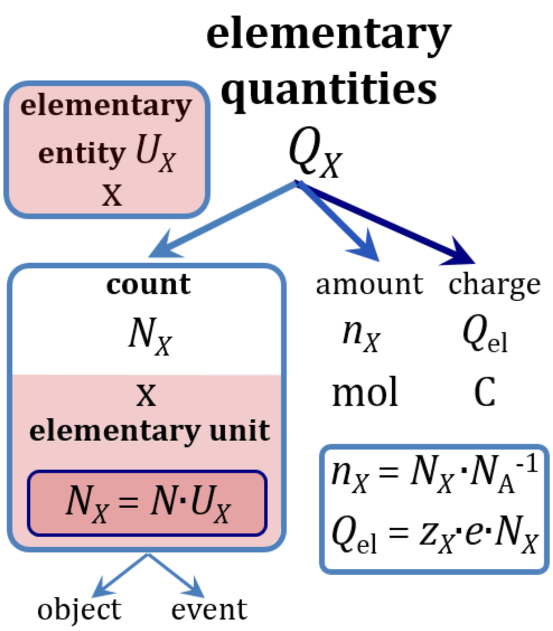
Let a biomedical scientist measure the body mass of patients on the third floor. Here the meaning of *X* is definitely elementary, and body mass is "mass per body". Let the same biomedical scientist take blood samples from each patient and move to the fourth floor for isolation of PBMC and platelets, taking a cell count, and measuring the cell mass. The meaning of *X*=body as perceived on the third floor (body mass M_{UX} = 60 kg·x^{-1}) switches subconsciously upon arrival on the fourth floor for studying respiration of living cells and *X*=ce, when cell mass is the mass measured in a sample containing a large number of cells (cell mass m_X = 6 mg). All this works practically and automatically very well even in a single person, as long as the quantities obtained on the patient's blood cells (metabolic oxygen consumption per mass of cells) are not directly related to a quantity obtained on the individual patient (V_{O2max}; respiratory capacity per individual, or $V_{O2max/M}$ per body mass).

Canonical recommendation

The BIPM and the Meter Convention (p. 118)

The 59 SI member states and 42 states and economies that were associates of the General Conference should consider to extend the number of 10 Consultative Committees, which presently cover:

Table 1. Count, amount, volume, mass normalized for count, amount, volume, mass. Canonical symbols adhere to a consistent formal system. Compared with the corresponding commonly used practical symbols, the canonical symbols help to decode the meaning of the quantities expressed by these symbols by showing their isomorphic form. For example, compare the Avogadro constant N_A and molar mass $M(X)$ with isomorphic canonical symbols N_{n_X} and M_{n_X}; or count concentration C_X, the inverse of the molar volume $V_m(X)^{-1}$, and density ρ_X with isomorphic canonical symbols N_{V_X}, n_{V_X}, and m_{V_X}, respectively.



Quantity	Unit	:	Normalized for quantity	Unit	=	Canonical symbol	Unit	Practical symbol	Quantity
count N_X	x	:	count N_X	x	=	1	1	-	-
amount n_X	mol	:	count N_X	x	=	n_{U_X}	mol·x^{-1}	N_A^{-1}	1/Avogadro constant
volume V_X	m^3	:	count N_X	x	=	V_{U_X}	m^3·x^{-1}		elementary volume
mass m_X	kg	:	count N_X	x	=	M_{U_X}	kg·x^{-1}		elementary mass
count N_X	x	:	amount n_X	mol	=	N_{n_X}	x·mol^{-1}	N_A	Avogadro constant
amount n_X	mol	:	amount n_X	mol	=	1	1	-	-
volume V_X	m^3	:	amount n_X	mol	=	V_{n_X}	m^3·mol^{-1}	$V_m(X)$	molar volume (IUPAC)
mass m_X	kg	:	amount n_X	mol	=	M_{n_X}	kg·mol^{-1}	$M(X)$	molar mass
count N_X	x	:	volume V_X	m^3	=	N_{V_X}	x·m^{-3}	C_X	number concentration* (IUPAC)
amount n_X	mol	:	volume V_X	m^3	=	n_{V_X}	mol·m^{-3}	$V_m(X)^{-1}$	1/molar volume
volume V_X	m^3	:	volume V_X	m^3	=	1	1	-	-
mass m_X	kg	:	volume V_X	m^3	=	m_{V_X}	kg·m^{-3}	ρ_X	density
count N_X	x	:	mass m_X	kg	=	N_{m_X}	x·kg^{-1}		
amount n_X	mol	:	mass m_X	kg	=	n_{m_X}	mol·kg^{-1}	$M(X)^{-1}$	1/molar mass
volume V_X	m^3	:	mass m_X	kg	=	V_{m_X}	m^3·kg^{-1}	v_X	specific volume (IUPAC)
mass m_X	kg	:	mass m_X	kg	=	1	1	-	-

* The term 'number concentration' is misleading and is to be replaced by 'count concentration' (see Number and count).

1. electricity and magnetism,
2. photometry and radiometry,
3. thermometry,
4. length,
5. time and frequency,
6. ionizing radiation,
7. units,
8. mass and related quantities,
9. amount of substance: metrology in chemistry and biology,
10. acoustics, ultrasound and vibration.

Insufficient care is taken to emphasize the fundamental role of the quantity *count*. A consistent symbol for the quantity *count* needs to be defined (**Table 1**). A unit and appropriate symbol for the unit of the quantity count needs to be implemented, such that a unit is not confused with a number, and a numeral is not used as a symbol for a unit (**Table 2**). To take these deficiencies into account, it is recommended to implement an 11th Consultative Committee on:

11. elementary entities and count.

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Conflict of interest
Erich Gnaiger is founder and CEO of Oroboros Instruments, and was chair of COST Action CA15203 MitoEAGLE (2016-2021).

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This preprint is part of a volume in preparation:
***X*-mass Carol. Body Mass Excess, mitObesity, and mitochondrial fitness.**

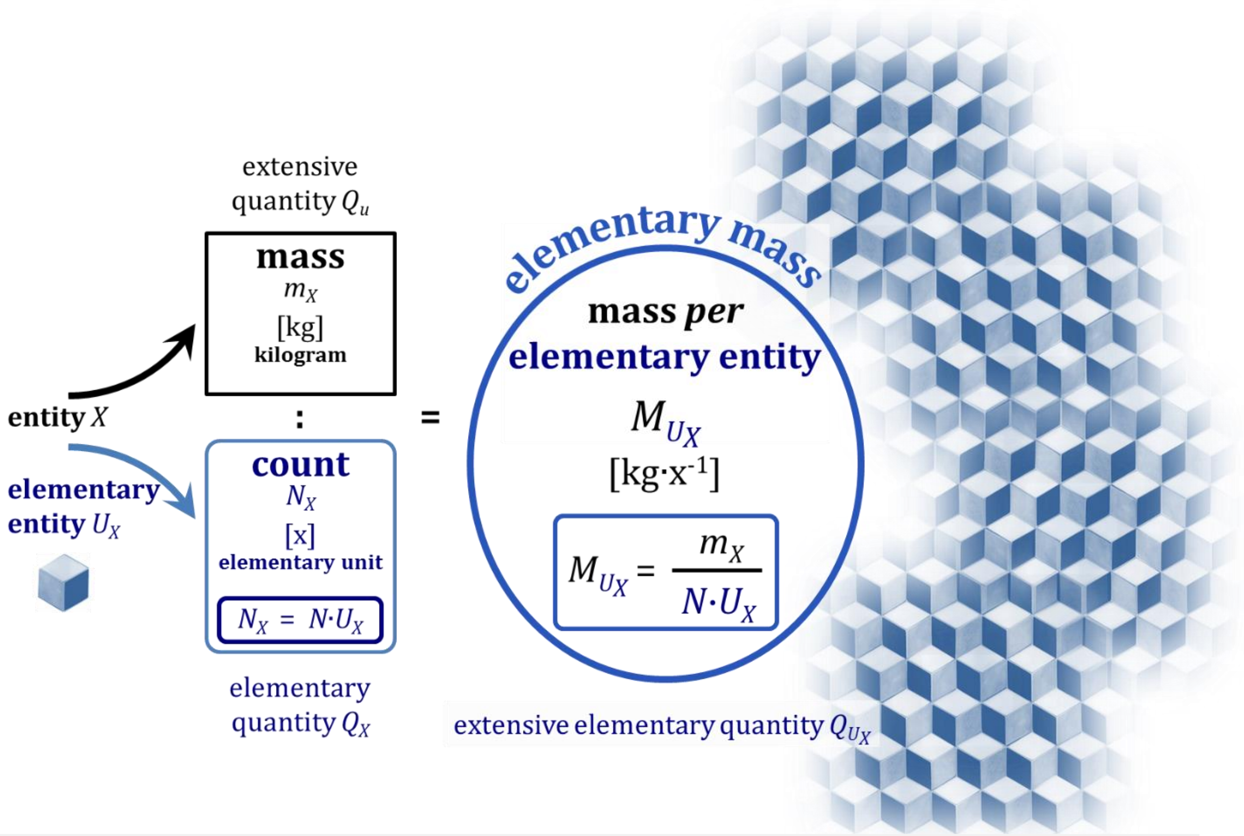


Figure 1. Elementary mass normalized for a count.

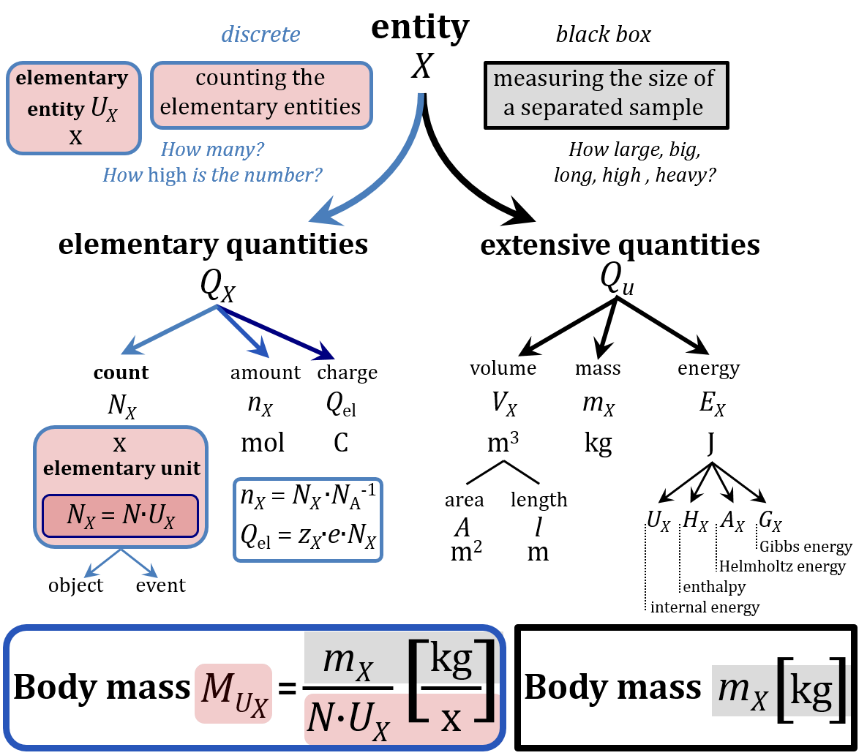
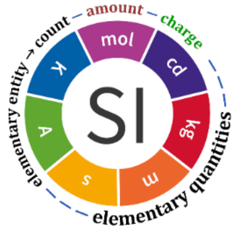


Figure 2. Elementary quantities stoichiometrically linked to counting of elementary entities (left) and extensive quantities based on measurements of a pure sample (right).

Table 2. List of SI base quantities and units extended by elementary entity, count, and charge. Combining elementary quantities in a table of fundamental quantities.



Quantity	Symbol for quantity Q	Symbol for dimension	Name of abstract unit u_Q	Symbol for unit u_Q [*]
elementary entity [*,$]	U_X	U	elementary unit	x
count [*,$]	$N_X = N \cdot U_X$	X	elementary unit	x
amount [*,§]	$n_X = N_X \cdot N_A^{-1}$	N	mole	mol
charge [*,€]	$Q_{el} = z_X \cdot e \cdot N_X$	I·T	coulomb	C = A·s
length	l	L	meter	m
mass	m	M	kilogram	kg
time	t	T	second	s
electric current	I	I	ampere	A
thermodynamic temperature	T	Θ	kelvin	K
luminous intensity	I_v	J	candela	cd

[*] SI units, except for the canonical 'elementary unit' [x]. The following footnotes are canonical comments.

[*] For the elementary quantities N_X, n_X, and Q_{el}, the entity-type X of the elementary entity U_X has to be specified in the text and indicated by a subscript: n_{O_2}; N_{ce}; Q_{el}.

[$] Count N_X equals the number of elementary entities U_X. In the SI, the quantity 'count' is explicitly considered as an exception: "Each of the seven base quantities used in the SI is regarded as having its own dimension. .. All other quantities, with the exception of counts, are derived quantities" (Bureau International des Poids et Mesures 2019 The International System of Units (SI)). An elementary entity U_X is a material unit, it is not a count (U_X is not a number of U_X). N_X has the dimension X of a count and U_X has the dimension U of an elementary entity; both quantities have the same abstract unit, the 'elementary unit' [x].

[§] Amount n_X is an elementary quantity, converting the elementary unit [x] into the SI base unit mole [mol] using the Avogadro constant N_A.

[€] Charge is a derived SI quantity. Charge is an elementary quantity, converting the elementary unit [x] into coulombs [C] using the elementary charge e, or converting moles [mol] into coulombs [C] using the Faraday constant F. z_X is the charge number per elementary entity U_X, which is a constant for any defined elementary entity U_X. $Q_{el} = z_X \cdot F \cdot n_X$

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